ADAPTIVE CONTROL OF UNDERACTUATED SYSTEMS IN SUBSPACE OF JOINT VARIABLES

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ABSTRACT

To eliminate the effect of backlash, shaft elasticity and friction in control of transmission system are conventionally nonlinear estimators for their compensations often used. This paper proposes a new design procedure of an adaptive and robust tracking controller for gearing mechanical transmission systems without using of such estimators and compensators. The proposed controller is designed by using the reduced order model of gearing transmission system. The asymptotic tracking behavior of the system in the presence of all uncertainties caused by backlash, friction or cogwheel elasticity is proved. The simulation results are provided to illustrate the satisfactory performance of the closed loop system.

TÓM TẮT

Để làm giảm được hiệu ứng của khe hở, moment xoắn và ma sát trong các hệ truyền động người ta thường sử dụng các thiết bị nhận dạng xấp xỉ các thành phần này rồi sau đó sẽ điều khiển bù những thành phần không mong muốn đó. Bài báo trình bày một phương pháp điều khiển hoàn toàn khác, vẫn làm giảm được hiệu ứng của những thành phần bất định đó mà không cần sử dụng thêm các khâu nhận dạng và điều khiển bù. Bộ điều khiển được thiết kế dựa trên mô hình hạ bậc của đối tượng. Khả năng tiệm cận tốt theo quỹ đạo mẫu của hệ có chứa đầy đủ các thành phần bất định đã được chứng minh cả về mặt lý thuyết và mô phỏng.

I. INTRODUCTION

The effect of immeasurable frictions, unpredictable elasticity of shafts and imprecise description of backlash is inevitable in mechanical transmission systems, which limits the performance of control system in practical applications. Those inevitable uncertainties can reduce the lifetime of the whole system or even disturb the system behavior. Therefore, damping the torsional vibration due to the shaft or cogwheel elasticity and suppressing the effect of friction or backlash are the most important control problems of mechanical systems in general and of gearing transmission systems in particular.

According to improve the control performance of gearing transmission systems have to be suppressed the frictions, shaft elasticity and backlash between cogwheel. The most promising approach for suppression of those uncertainties can be obtained with estimators of backlash, shaft elasticity and frictions and then compensate these by states feedback control [1,3,4,5].

Such proposed control strategy, however, can only be used good either for systems with shaft elasticity or with backlash separately [6]. Moreover, a good tracking performance of systems, in which all uncertainties like immeasurable friction, unpredictable elasticity of shafts and backlash are simultaneous present, cannot be achieved with such compensative techniques.

To overcome this problem is an avoidance of estimators and compensators included in controller necessary. Therefore will be now applied here an adaptive robust control based on the sliding mode technique and the certainty equivalence principle, which allows to improve the overall tracking performance of the closed loop system without using of additional estimators and compensators for elimination of uncertainties in systems.

The sliding mode control is one of the robust control theories to suppress the effect of bounded noises or disturbances in systems. In addition, the certainty equivalence is also the most successfully used principle in adaptive controller designs for uncertain nonlinear systems in the presence of unknown constants in the systems' model. In this connection, the paper combines both the sliding mode technique and the certainty equivalence principle for designing an adaptive robust tracking controller for gearing transmission systems, in which the unpredictable elasticity of cogwheels and the imprecise description of backlash between cogwheels are considered as unknown constant parameters, whereas immeasurable shaft friction and the load capacity are regarded as bounded time dependent noises and disturbances in the system.

This paper is organized as follows. In section 2 is included the reduced order model of gearing transmission systems. The section 3 describes design procedure of robust adaptive controller for system based on this reduced order model. In section 4 the experimental results and simulations are includes. Finally are included in section 5 some conclutions and commentaries about future research.

II. MODELLING OF GEARING TRANSMISSION SYSTEMS

Consider a gearing transmission system with a controller depicted in Figure 1. The driving motor provides a control torque M_d which is transmitted to the load M_c through two wheel gears and two elastic shafts.

Let M_{f1} and M_{f2} denote the friction moment on each shaft. Both shafts have the same elasticity factor denoted by c. Let φ_1 and φ_2 be the rotational angles of corresponding shaft and α the backlash between cogwheels. The Euler-Lagrange model of this gearing transmission system is given as follows (see, for example, [2]).

$$\begin{cases} \bar{J}_1\ddot{\varphi}_1 + cr_1^2\cos^2\alpha(\varphi_1 + i_{12}\varphi_2) = M_d - M_{f1} \\ J_2\ddot{\varphi}_2 - cr_2^2\cos^2\alpha(\varphi_2 + i_{21}\varphi_1) = -M_c - M_{f2} \end{cases} \quad (1)$$

where r_1 and r_2 are the outer radii of corresponding wheels 1 and 2, $i_{12} = i_{21}^{-1}$ is the transmission rate of the two wheels and J_1, J_2, J_d are the inertia moments of wheel 1, wheel 2 and the driving motor respectively and $\bar{J}_1 = J_d + J_1$ denotes the sum of inertia moments of wheel 1 and the driving motor.

While $\bar{J}_1$, J_2, i_{12}, i_{21}, r_1 and r_2 in Euler-Lagrange model (1) can be considered as known parameters, the other parameters such as shaft elasticity c, friction moments M_{f1}, M_{f2}, load moment M_c, backlash α are all uncertainties or disturbances of the system. Therefore in the following, all unknown constant parameters of the model will be denoted by θ_k, whereas disturbances by d_k.

By using

$$\begin{aligned} \theta_1 &= cr_1^2\cos^2\alpha \\ \theta_2^{-1} &= cr_2^2\cos^2\alpha \\ M_{f1} &= b_1\dot{\varphi}_1 + d_1(\bar{\varphi}_1, t) \\ M_c - M_{f2} &= -b_2\dot{\varphi}_2 - d_2(\bar{\varphi}_2, t) \end{aligned} \quad (2)$$

where

$b_1,\ b_2$ – known constants,

$\theta_1,\ \theta_2$ – unknown constants,

$\bar{\varphi}_1 = \left(\varphi_1\ ,\dot{\varphi}_1\ ,\ddot{\varphi}_1\ ,\ \dots\ ,\varphi_1^{(p)}\right)^T$,

$\bar{\varphi}_2 = \left(\varphi_2\ ,\dot{\varphi}_2\ ,\ddot{\varphi}_2\ ,\ \dots\ ,\varphi_2^{(q)}\right)^T$,

$x^{(k)}$ – k^{th} derivative of x,

$p,\ q$ – finite positive integers,

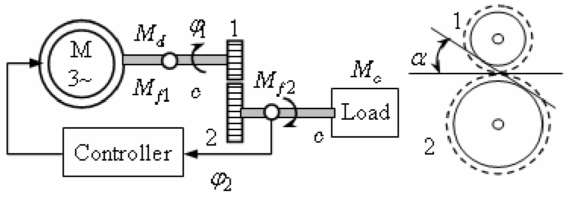

Figure 1. Configuration of a gearing transmission system

$d_1(\bar{\varphi}_1, t)$, $d_2(\bar{\varphi}_2, t)$ – unknown disturbances, the Euler-Lagrange model (1) becomes

$$\begin{cases} \bar{J}_1\ddot{\varphi}_1 + \theta_1(\varphi_1 + i_{12}\varphi_2) = M_d - b_1\dot{\varphi}_1 - d_1 \\ J_2\ddot{\varphi}_2 - \theta_2^{-1}(\varphi_2 + i_{12}^{-1}\varphi_1) = -b_2\dot{\varphi}_2 - d_2 \end{cases} \quad (3)$$

From the second equation of (3), it is easy to see that

$$\begin{aligned} \varphi_1 &= i_{12}\left[\theta_2\left(J_2\ddot{\varphi}_2 + b_2\dot{\varphi}_2 + d_2\right) - \varphi_2\right] \\ &= \theta_3\ddot{\varphi}_2 + \theta_4\dot{\varphi}_2 - i_{12}\varphi_2 + d_3 \end{aligned} \quad (4)$$

with

$$d_3 = i_{12}\theta_2 d_2,\ \ \theta_3 = \theta_2 J_2,\ \ \theta_4 = i_{12}\theta_2 b_2$$

and

$$\begin{aligned} \dot{\varphi}_1 &= \theta_3\dddot{\varphi}_2 + \theta_4\ddot{\varphi}_2 - i_{12}\dot{\varphi}_2 + d_4 \\ \ddot{\varphi}_1 &= \theta_3\varphi_2^{(4)} + \theta_4\dddot{\varphi}_2 - i_{12}\ddot{\varphi}_2 + d_5 \end{aligned} \quad (5)$$

where

$d_4 = \dot{d}_3$, $d_5 = \dot{d}_4$.

From (3), (4) and (5), it follows that

$$\begin{aligned} M_d = {} & \bar{J}_1\theta_3\varphi_2^{(4)} + \left(\bar{J}_1\theta_4 + b_1\theta_3\right)\dddot{\varphi}_2 + \\ & + \left(b_1\theta_4 + \theta_1\theta_3 - \bar{J}_1 i_{12}\right)\ddot{\varphi}_2 + \\ & + \left(\theta_1\theta_4 - b_1 i_{12}\right)\dot{\varphi}_2 + \\ & + \left(\bar{J}_1 d_5 + \theta_1 d_3 + b_1 d_4 + d_1\right) \end{aligned}$$

Next, let states vector $\boldsymbol{x}$, truncated states vector $\bar{\boldsymbol{x}}$, input control signal u, vector of unknown constants $\boldsymbol{\theta}_f$, unknown constant θ_g and unknown disturbance $d(\boldsymbol{x},t)$ be defined as follows

$$\boldsymbol{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} \varphi_2 \\ \dot{\varphi}_2 \\ \ddot{\varphi}_2 \\ \dddot{\varphi}_2 \end{pmatrix},\ \ \bar{\boldsymbol{x}} = \begin{pmatrix} x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} \dot{\varphi}_2 \\ \ddot{\varphi}_2 \\ \dddot{\varphi}_2 \end{pmatrix},\ \ u = M_d$$

$$\boldsymbol{\theta}_f = -\frac{1}{\bar{J}_1\theta_3}\begin{pmatrix} \theta_1\theta_4 - b_1 i_{12} \\ b_1\theta_4 + \theta_1\theta_3 - \bar{J}_1 i_{12} \\ \bar{J}_1\theta_4 + b_1\theta_3 \end{pmatrix},\ \ \theta_g = \bar{J}_1\theta_3$$

$$d(\boldsymbol{x},t) = -\frac{1}{\bar{J}_1\theta_3}\left(\bar{J}_1 d_5 + \theta_1 d_3 + b_1 d_4 + d_1\right)$$

The Euler-Lagrange model (3) of the gearing transmission system, can now be rewritten in the form of uncertain states fourth order model:

$$\begin{cases} \dot{x}_k = x_{k+1} \ \text{ if } \ 1 \le k \le 3 \\ \dot{x}_4 = \boldsymbol{\theta}_f^T\bar{\boldsymbol{x}} + d(\boldsymbol{x},t) + \theta_g u \end{cases} \quad (6)$$

with $d(\boldsymbol{x},t)$ being bounded by a number $\delta > 0$, that is

$$|d(\boldsymbol{x},t)| \le \delta \ \text{ for all } \boldsymbol{x},t . \quad (7)$$

Because reference signal $w(t)$ is often the desired speed $\dot{\varphi}_2 = x_2$, not the the rotational angle $\varphi_2 = x_1$, the reduced order model:

$$\begin{cases} \dot{x}_2 = x_3 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \boldsymbol{\theta}_f^T\bar{\boldsymbol{x}} + d(\boldsymbol{x},t) + \theta_g u \end{cases} \quad (8)$$

can be employed in controller design instead of fourth order model (6). Furthermore the using of reduced order model (8) will definitely guarantee for obtaining of a more simple controller than by using of original fourth order model (6).

III. CONTROLLER DESIGN

Let be first $w(t)$ the reference signal, so the reference trajectory for reduced order system (8) will be:

$$\boldsymbol{w} = \left(w\ ,\ \dot{w}\ ,\ \ddot{w}\right)^T \quad (9)$$

and the vector of reference error is

$$\boldsymbol{e} = \left(e\ ,\ \dot{e}\ ,\ \ddot{e}\right)^T \quad (10)$$

where

$$e = w - x_2 = w - \dot{\varphi}_2 \quad (11)$$

To control the states vector $\boldsymbol{x}(t)$ of system (8) to asymptotically track the reference trajectory $\boldsymbol{w}(t)$ given in (9) based on sliding mode control, first the following sliding surface is used:

$$s(e) = a_1 e + a_2\dot{e} + \ddot{e} = \boldsymbol{a}^T\boldsymbol{e} \quad (12)$$

where all elements $a_1,\ a_2$ of vector

$$\boldsymbol{a}^T = \left(a_1\ ,\ a_2\ ,\ 1\right)^T$$

are chosen such that the following polynomial

$$p(\alpha) = a_1 + a_2\alpha + \alpha^2 \quad (13)$$

will be Hurwitz. Because polynomial (13) is from second order, necessary and sufficient for Hurwitzian of $p(\alpha)$ is $a_1 > 0$ and $a_2 > 0$.

Moreover, by using sliding surface given in (12), in order to ensure the asymptotic tracking performance

$\boldsymbol{e} \to \boldsymbol{0}$ and $|e| < \infty$

the nesecessary and sufficient condition is

$s(e) = 0$

Thus, the initial tracking control aim can now be replaced with

$s(e) \to 0$ and $|s(e)| < \infty$ for $t > 0$

Now consider the following candidate control Lyapunov function (CLF)

$$V = \frac{1}{2} s(e)^2$$

with its derivative being given by

$$\begin{aligned} \dot{V} &= s\dot{s} = s\left(a_1\dot{e} + a_2\ddot{e} + \dddot{w} - \dot{x}_4\right) \\ &= s\left(a_1\dot{e} + a_2\ddot{e} + \dddot{w} - \boldsymbol{\theta}_f^T \overline{\boldsymbol{x}} - d(\boldsymbol{x},t) - \theta_g u\right) \end{aligned}$$

Therefore, if the following controller is used

$$u = \theta_g^{-1}\left(a_1\dot{e} + a_2\ddot{e} + \dddot{w} - \boldsymbol{\theta}_f^T \overline{\boldsymbol{x}} + \lambda \operatorname{sgn}(s)\right) \tag{14}$$

with any $\lambda > \delta$ then

$$\begin{aligned} \dot{V} &= s\left(a_1\dot{e} + a_2\ddot{e} + \dddot{w} - \boldsymbol{\theta}_f^T \overline{\boldsymbol{x}} - d(\boldsymbol{x},t) - \theta_g u\right) \\ &= -sd(\boldsymbol{x},t) - \lambda s \operatorname{sgn}\left(s(e)\right) \\ &\le |s||d(\boldsymbol{x},t)| - \lambda s \operatorname{sgn}\left(s(e)\right) \\ &\le \left(\delta - \lambda\right)|s| < 0, \end{aligned}$$

which sufficiently ensures the boundedness of $|s(e)|$ as well as the asymptotic decay to zero of $s(e)$.

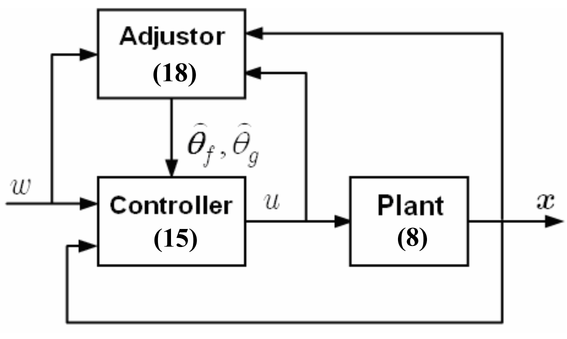

Figure 2. Configuration of the closed loop system

In practice, the controller (14), however, cannot be used because of the unknown constant vector $\boldsymbol{\theta}_f$ and unknown parameter θ_g. To overcome this limitation, the certainty equivalence principle will be employed.

First, the unknown constants $\boldsymbol{\theta}_f$ and θ_g in (14) are replaced by time functions $\hat{\boldsymbol{\theta}}_f(t)$ and $\hat{\theta}_g(t)$, respectively, yielding

$$u = \frac{a_1\dot{e} + a_2\ddot{e} + \dddot{w} - \hat{\boldsymbol{\theta}}_f^T \overline{\boldsymbol{x}} + \lambda \operatorname{sgn}(s)}{\hat{\theta}_g} \tag{15}$$

with any chosen parameter $\lambda > \delta$.

With this replacement, the derivative of the sliding surface (12) is now given by

$$\begin{aligned} \dot{s} &= a_1\dot{e} + a_2\ddot{e} + \dddot{w} - \dot{x}_4 \\ &= a_1\dot{e} + a_2\ddot{e} + \dddot{w} - \left[\boldsymbol{\theta}_f^T \overline{\boldsymbol{x}} + d(\boldsymbol{x},t) + \theta_g u\right] \\ &= a_1\dot{e} + a_2\ddot{e} + \dddot{w} - \\ &\qquad - \left[\boldsymbol{\theta}_f^T \overline{\boldsymbol{x}} + d(\boldsymbol{x},t) + \left(\theta_g - \hat{\theta}_g\right)u + \hat{\theta}_g u\right] \\ &= \left(\hat{\boldsymbol{\theta}}_f - \boldsymbol{\theta}_f\right)^T \overline{\boldsymbol{x}} + \left(\hat{\theta}_g - \theta_g\right)u - d(\boldsymbol{x},t) - \lambda \operatorname{sgn}(s) \\ &= \boldsymbol{\delta}_f^T \overline{\boldsymbol{x}} + \delta_g u - d(\boldsymbol{x},t) - \lambda \operatorname{sgn}(s) \end{aligned}$$

where

$\boldsymbol{\delta}_f = \hat{\boldsymbol{\theta}}_f - \boldsymbol{\theta}_f$ and $\delta_g = \hat{\theta}_g - \theta_g$.

It can be noted further that

$$\dot{\boldsymbol{\delta}}_f = \dot{\hat{\boldsymbol{\theta}}}_f \quad \text{and} \quad \dot{\delta}_g = \dot{\hat{\theta}}_g. \tag{16}$$

Second, by using an adaptive CLF candidate

$$\hat{V} = \frac{1}{2}s^2 + \frac{1}{2}\boldsymbol{\delta}_f^T \mathbf{F}^{-1} \boldsymbol{\delta}_f + \frac{1}{2\xi}\delta_g^2 \tag{17}$$

where $\mathbf{F} \in \mathbb{R}^{3\times3}$ is any symmetric positive definite matrix and ξ is an arbitrary positive constant.

By using the derivative of the sliding surface $\dot{s}$ and the fact (16), one subsequently obtains directly from (17)

$$\begin{aligned} \dot{\hat{V}} &= s\dot{s} + \boldsymbol{\delta}_f^T \mathbf{F}^{-1} \dot{\boldsymbol{\delta}}_f + \frac{1}{\xi}\delta_g \dot{\delta}_g \\ &= s\left[\boldsymbol{\delta}_f^T \overline{\boldsymbol{x}} + \delta_g u - d(\boldsymbol{x},t) - \lambda \operatorname{sgn}(s)\right] \\ &\qquad + \boldsymbol{\delta}_f^T \mathbf{F}^{-1} \dot{\hat{\boldsymbol{\theta}}}_f + \frac{1}{\xi}\delta_g \dot{\hat{\theta}}_g \\ &= -sd(\boldsymbol{x},t) - s\lambda \operatorname{sgn}(s) + \boldsymbol{\delta}_f^T \left(s\overline{\boldsymbol{x}} + \mathbf{F}^{-1}\dot{\hat{\boldsymbol{\theta}}}_f\right) + \\ &\qquad + \delta_g\left(su + \frac{1}{\xi}\dot{\hat{\theta}}_g\right) \end{aligned}$$

Now, by using the following adaptive adjustments for the time functions $\widehat{\boldsymbol{\theta}}_f(t)$ and $\widehat{\theta}_g(t)$ of controller (15)

$$\begin{cases} \dot{\widehat{\boldsymbol{\theta}}}_f = -\mathbf{F}\, s(e)\overline{\boldsymbol{x}} \\ \dot{\widehat{\theta}}_g = -\xi s(e) u \end{cases} \qquad (18)$$

the derivative $\dot{V}$ becomes negative definite

$$\dot{V} = -sd(\boldsymbol{x},t) - s\lambda\,\mathrm{sgn}(s) \le |s||d(\boldsymbol{x},t)| - \lambda|s|$$
$$\le (\delta - \lambda)|s| < 0$$

which is sufficient for ensuring that $|s(e)| < \infty$ and $s(e) \to 0$

Figure 2. shows the main configuration of the closed loop system, in which the designed controller, including sliding mode controller (15) and adaptive parameters laws (18), always drives the output $y = x_2 = \dot{\varphi}_2$ asymptotically convergent to any three times differentiable desired trajectory $w(t)$.

IV. NUMERICAL EXAMPLE

Consider a gearing transmission system with reduced order states model (8), where $d(\boldsymbol{x},t)$ is a white noise with $\|d\|_\infty = 0.1$. Let design parameters be chosen as follows:

$$\mathbf{F} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}, \ \xi = 0.1, \ \lambda = 0.9, \ a_1 = a_2 = 1.8$$

The tracking error and the system output are shown in Figure 3. , the vector $\widehat{\boldsymbol{\theta}}_f$ and $\widehat{\theta}_g$ from the adaptive adjustors, are also given in Figure 4. and in Figure 7. -Figure 7. respectively.

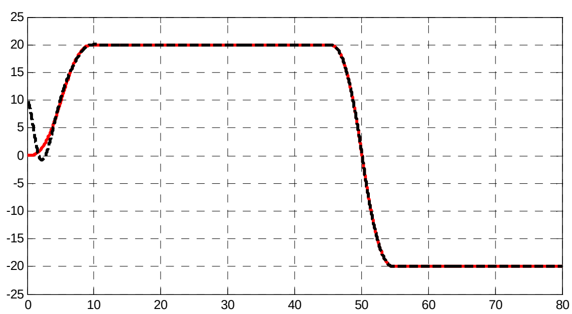

Figure 3. Desired trajectory and system output

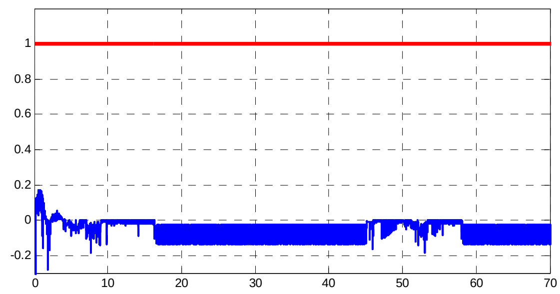

Figure 4. Adjusted parameter $\widehat{\theta}_g$ compared with θ_g

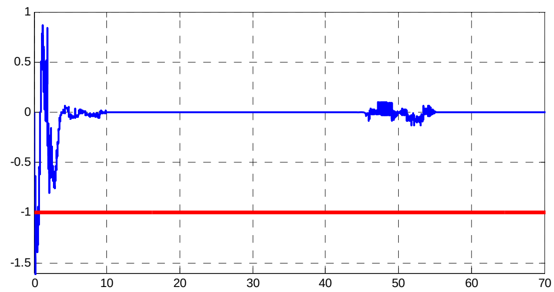

Figure 5. Adjusted $\widehat{\boldsymbol{\theta}}_f[1]$ compared with $\boldsymbol{\theta}_f[1]$

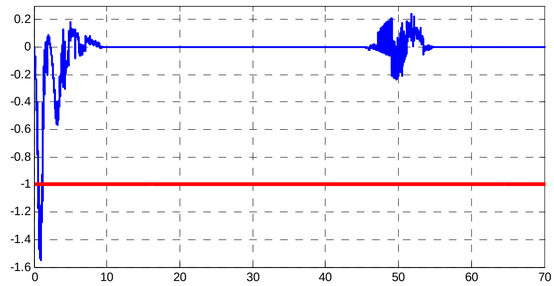

Figure 6. Adjusted $\widehat{\boldsymbol{\theta}}_f[2]$ compared with $\boldsymbol{\theta}_f[2]$

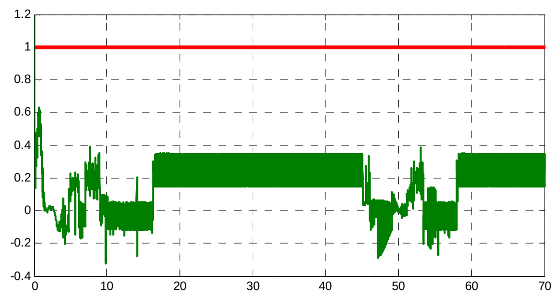

Figure 7. Adjusted $\widehat{\boldsymbol{\theta}}_f[3]$ compared with $\boldsymbol{\theta}_f[3]$

From the simulation results, it can be seen that the system output asymptotically converges to the desired trajectory even in the presence of the unknown parameters $\boldsymbol{\theta}_f$, θ_g and the bounded disturbance $d(\boldsymbol{x},t)$. It should be noted that the adjusted parameters $\widehat{\boldsymbol{\theta}}_f$ and $\widehat{\theta}_g$ do not tend to the actual values of unknown parameters $\boldsymbol{\theta}_f$ and θ_g. In fact, in this example, the plant (8) was simulated with

$$\boldsymbol{\theta}_f = (1\ ,\ -1\ ,\ -1)^T\ ,\ \ \theta_g = 1$$

However, this does not affect the tracking performance of the system.

V. CONCLUSION

The adaptive and robust controller for gearing transmission system depicted in Figure 1. , which is designed by using the procedure proposed in this paper based on the reduced order model (8), obviously satisfies the tracking requirement of the system. This satisfaction has been proved theoretically and numerically. In fact, the controller can effectively attenuate the disturbance and suppess the effect of parameter uncertainties.

Note that although the tracking error is guaranteed to be zero at its steady state, its value during the transient period cannot be constrained in a predetermined range. This limitation can be avoided by using a barrier CLF instead of (17) and choosing a_1, a_2 of the sliding surface (12) appropriately.

Furthermore, as a consequence of using sliding mode control, there still exists the chattering in the system. In oder to damp this undesired behavior, the constant λ should be chosen as small as possible but not less than δ.

In the case, that the constant λ has to be choosen less than δ, the controller (15) can be revised as

$$u = \frac{a_1\dot{e} + a_2\ddot{e} + \dddot{w} - \widehat{d}(\boldsymbol{x},t) - \widehat{\boldsymbol{\theta}}_f^T \overline{\boldsymbol{x}} + \lambda\,\mathrm{sgn}(s)}{\widehat{\theta}_g}$$

where $\widehat{d}(\boldsymbol{x},t)$ is an estimate of $d(\boldsymbol{x},t)$ such that

$$\sup_{\boldsymbol{x},t} \left| \widehat{d}(\boldsymbol{x},t) - d(\boldsymbol{x},t) \right| < \lambda < \delta$$

The function $\widehat{d}(\boldsymbol{x},t)$ can be obtained easily by using, for example, a neural network or fuzzy system.

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